Collective phenomena in semiconductor microcavities

Exciton-Polaritons. Brief overview

Outline

1°) Microcavities and cavity polaritons.

2°) Polarization and spin of cavity polaritons.

3°) Polariton BEC and superfluidity

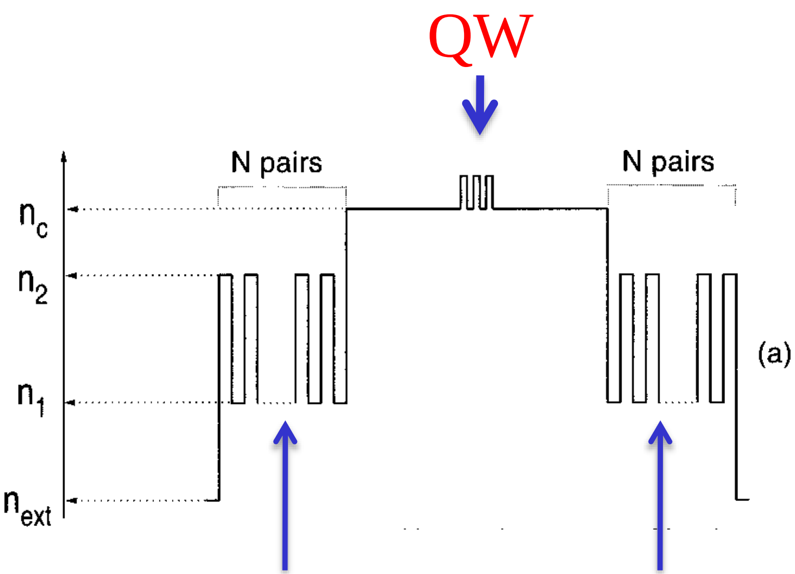
4°) Bistability and multistability phenomena in microcavities

5°) Polariton- induced superconductivity.

Quantum microcavities

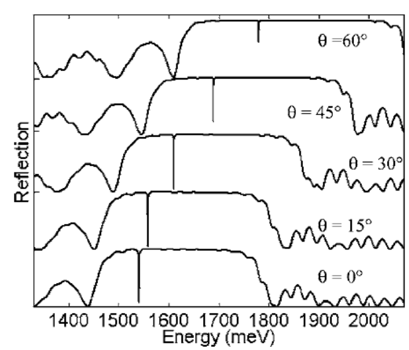
How can we increase light- matter coupling?

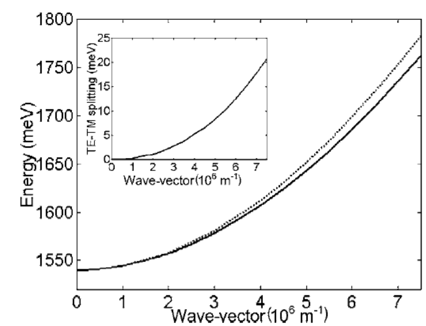
Structure of a microcavity



Bragg mirrors

The role of the Bragg mirrors: confinement of electromagnetic field. For infinite number of the layers the photonic bandgap is formed in the mirrors, and photon becomes perfectly confined inside a cavity.



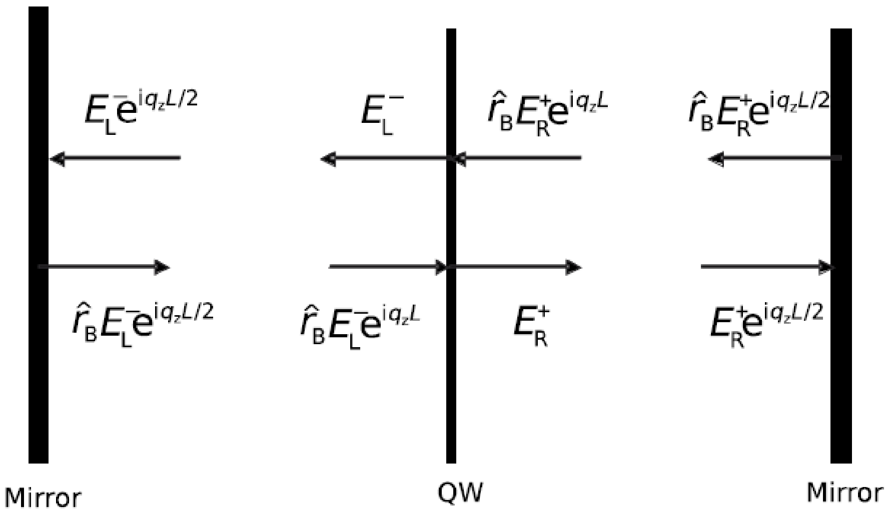


Confined photon aquires a mass

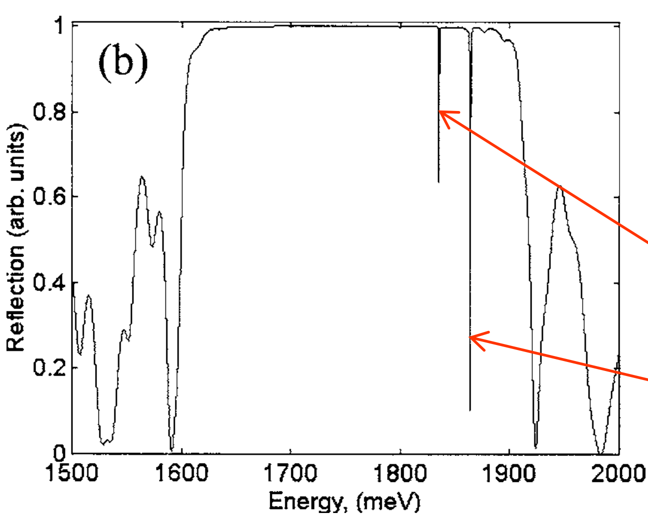
$$\hbar\omega_c = \hbar\frac{c}{n_c}\sqrt{q^2+k_z^2} \approx \hbar\frac{c}{n_c}k_z\left(1+\frac{q^2}{2k_z^2}\right) \equiv \frac{hc}{n_cL_c}+\frac{\hbar^2q^2}{2m_{ph}}$$

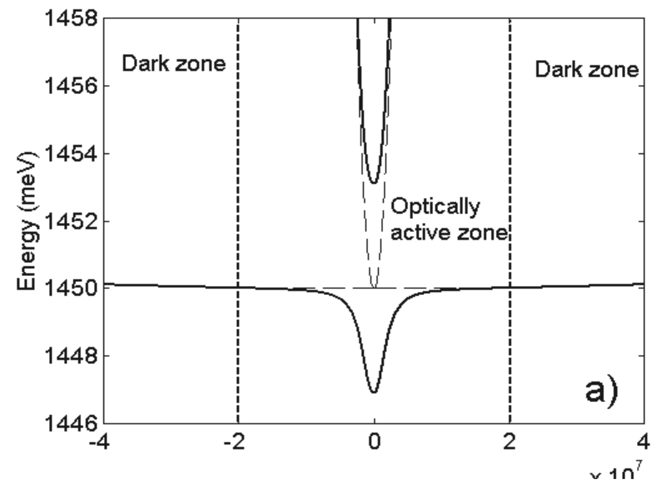
$$m_{ph} = \frac{hn_c}{cL_c} \sim 10^{-4} \div 10^{-5}\, m_e$$

Quantum microcavities with embedded QWs



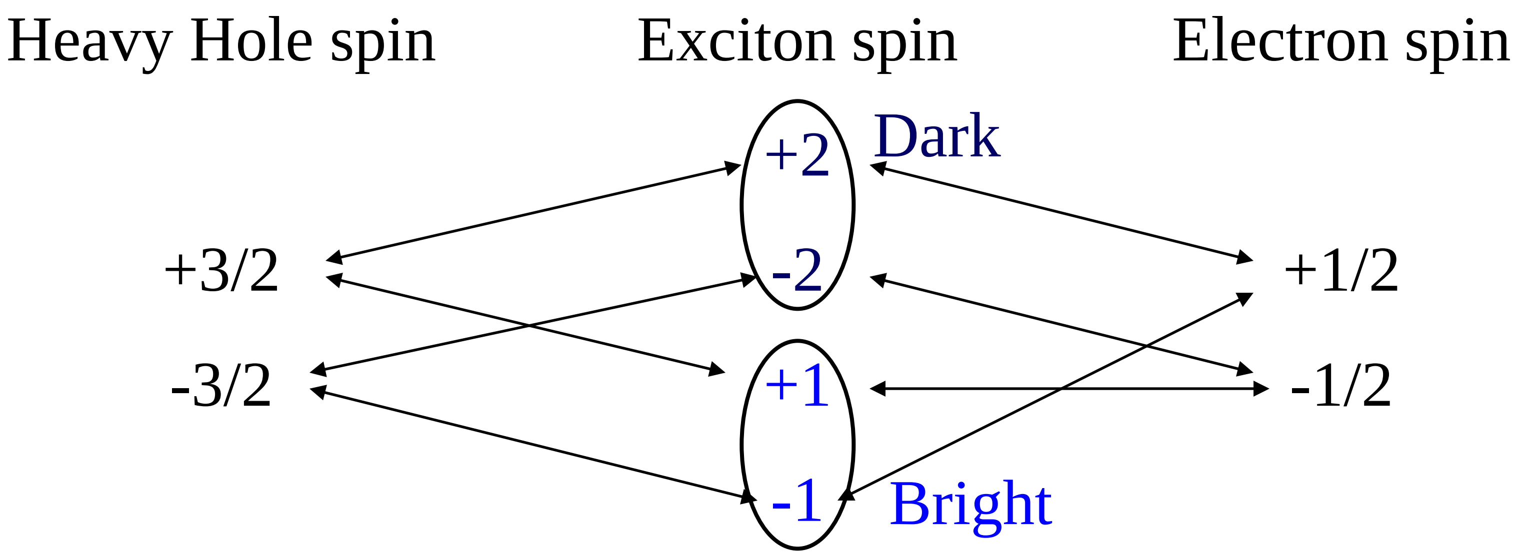
$$\begin{vmatrix} \dfrac{\hbar^2 k^2}{2m_{ex}} - E(k) & V/2 \\ V/2 & \dfrac{\hbar^2 k^2}{2m_{ph}} + \delta - E(k) \end{vmatrix} = 0$$





In a reflection spectra one has two dips instead of a single one

Spin structure of quantum well excitons and cavity polaritons.



1°) **Elliot Yaffet:** Hole spin flip. $+2 \leftrightarrow -1 \quad -2 \leftrightarrow +1$

2°) **D'yakonov Perel:** Electron spin flip due to spin orbit interaction. $+2 \leftrightarrow +1 \quad -2 \leftrightarrow -1$

3°) **Bir Aronov Pikus:** spin flip exchange interaction of electrons and holes. $+1 \leftrightarrow -1$

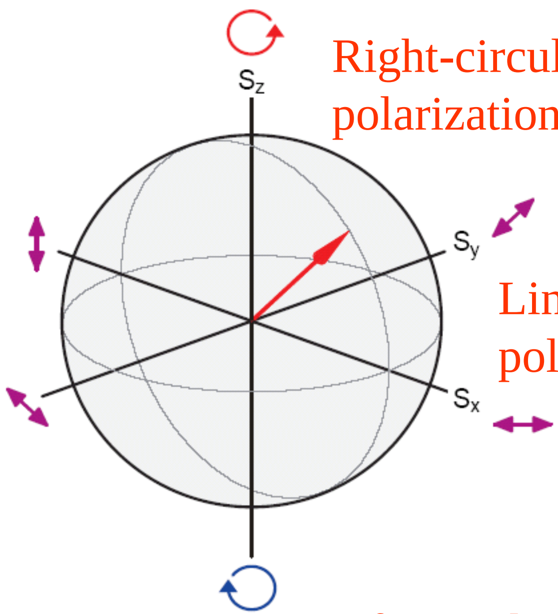
\+ **nonlinear effects**

Pseudospin and Poincare sphere

We take into account only +1 and -1 states (two-component wave function) which can be represented as a pseudo spin vector lying on the Poincaré sphere.

Density matrix of a two level system

$$\rho_{\vec{k}} = \frac{N_{\vec{k}}}{2} I + \vec{S}_{\vec{k}}.\sigma = \begin{pmatrix} N_{\vec{k}}/2 + S_{\vec{k},z} & S_{\vec{k},x} - iS_{\vec{k},y} \\ S_{\vec{k},x} + iS_{\vec{k},y} & N_{\vec{k}}/2 - S_{\vec{k},z} \end{pmatrix}$$

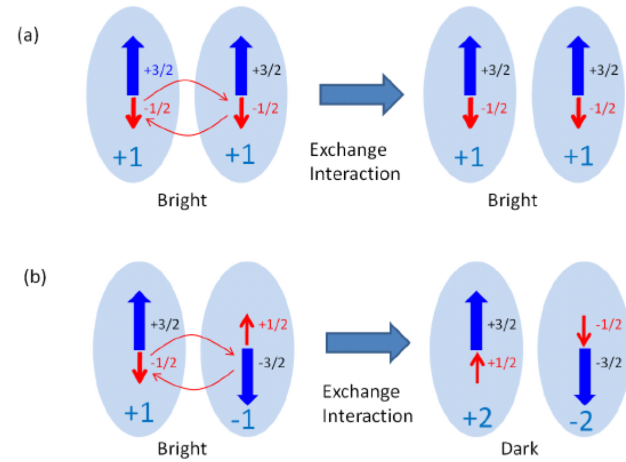


Right-circular polarization

Linear polarizations

Left-circular polarization

Polariton-polariton interactions are spin-anisotropic:



$$n = n_{\uparrow} - n_{\downarrow}; \quad S_z = \frac{n_{\uparrow} - n_{\downarrow}}{2}$$

Imbalance between spin populations provokes an energy splitting between z- polarized polaritons

$$\Delta E = (\alpha_1 - \alpha_2)(n_{\uparrow} - n_{\downarrow})$$

How microcavities can be used?

- Polaritons are bosons- BEC
- Polaritons interact with each other-superfluidity and nonlinear phenomena (multistability)
- Polaritons have “spin”- spinoptronics applications
- Polariton- mediated superconductivity
- Novel sources of THz radiation

Which particles to condense?

"Each mathematician is a special case, and in general mathematicians tend to behave like "fermions" i.e. avoid working in areas which are too trendy whereas physicists behave a lot more like "bosons" which coalesce in large packs andare often "overselling" their doings, an attitude which mathematicians despise"

A. Connes, French mathematician

Only composite particles- the "elementary" bosons are quite exotic.

Candidates:

1. Atoms with integer spin- e.g. ^{4}He, ^{86}Rb
2. Excitons- bound electron- hole pairs
3. Exciton polaritons (explanations later…)

Shortcoming:

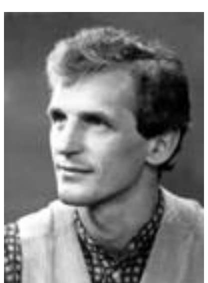
The particle should be a "good boson"

$$\left[b^{+};b\right]=1+O\left(na_{B}^{3}\right)$$

The system should be diluted!

"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"


Eric A. Cornell


Wolfgang Ketterle


Carl E. Wieman

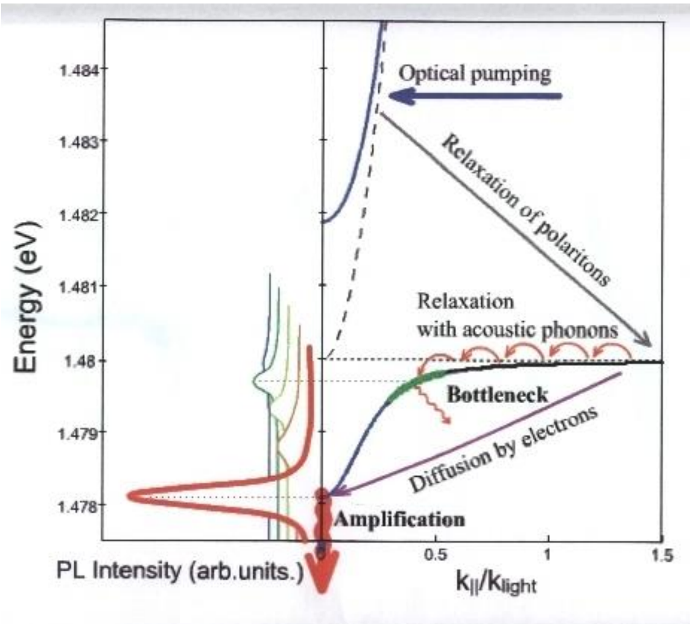
The Nobel Prize in Physics 2001

$$T_c \approx \frac{3.3\hbar^2}{mk_B} n^{2/3}$$

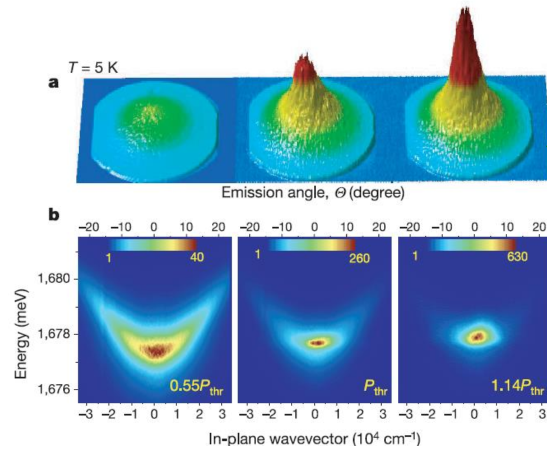
$$T_c << 1K$$

Critical temperature is inversly proportional to the mass of the condensing particles.

Polariton BEC

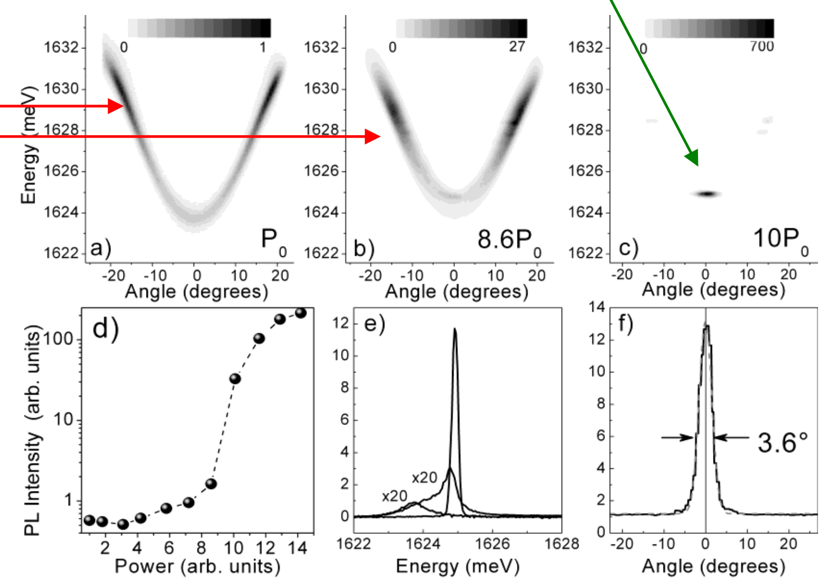


The group of Le Si Dang claimed the **existence of the polariton BEC** in Cd Te cavity at 20K (Kasprzak et al. Nature, 2006)



High pump: BEC

Low pump: bottleneck



Small effective mass, and consequently one can expect high critical temperature of BEC and critical superfluid velocity

$$T_c \approx \frac{3.3\hbar^2}{mk_B} n^{2/3} \qquad v_s = \sqrt{\frac{Un}{m}}$$

GaN cavities: experimental evidence of polariton BEC at room temperature

Superfluidity of cavity polaritons

$$H_{\text{int}} = \frac{U}{2V} \sum_{\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_3+\mathbf{p}_4} b^+_{\mathbf{p}_4} b^+_{\mathbf{p}_3} b_{\mathbf{p}_2} b_{\mathbf{p}_1} \approx \frac{UN^2}{2V} + \sum_{p\neq 0}\left[\left(E_0(k)+\frac{UN}{V}\right)\left(b^+_{\mathbf{p}} b_{\mathbf{p}} + b^+_{\text{-}\mathbf{p}} b_{\text{-}\mathbf{p}}\right) + \frac{UN}{V}\left(b^+_{\mathbf{p}} b^+_{\text{-}\mathbf{p}} + b_{\mathbf{p}} b_{\text{-}\mathbf{p}}\right)\right]$$

$$b_{\mathbf{p}} = \frac{1}{\sqrt{1-\left|A_{\mathbf{p}}\right|^2}}\left(\beta_{\mathbf{p}} + A_{\mathbf{p}}\beta^+_{-\mathbf{p}}\right) \qquad b^+_{\mathbf{p}} = \frac{1}{\sqrt{1-\left|A_{\mathbf{p}}\right|^2}}\left(\beta^+_{\mathbf{p}} + A_{\mathbf{p}}\beta_{-\mathbf{p}}\right)$$

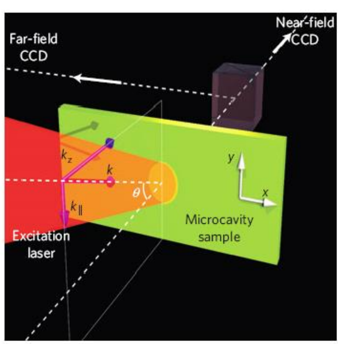
$$v_s = \sqrt{\frac{Un}{m}}$$

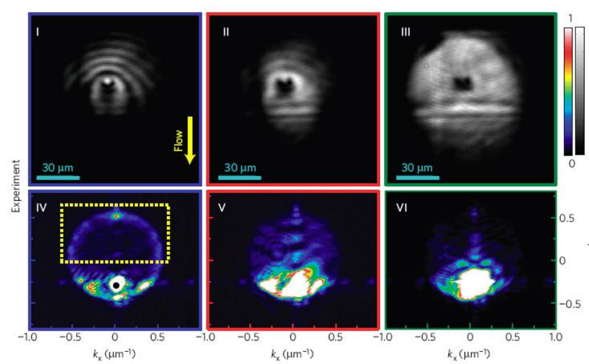
$$E_b(k) = \sqrt{E_0^2(k) + 2UnE_0(k)} \approx \hbar k v_s, \quad k \to 0$$

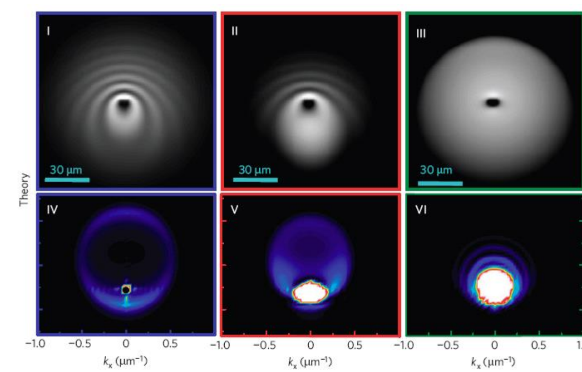
A.Amo et al, Nature 2009

A.Amo et al, Nature Phys. 2009

The smaller the mass the better!







Polarization of the condensate and dispersion of its elementary excitations for polariton BEC

Polarization of the condensate can be found by the minimization of the density of Free energy on pseudospin sphere

$$H = -2\Omega S_z + \frac{1}{4}(\alpha_1 + \alpha_2) n^2 + (\alpha_1 - \alpha_2) S_z^2$$

$$\left(\frac{n^2}{4} - S_z^2\right)\left(\Omega_z - (\alpha_1 - \alpha_2) S_z\right)^2 = 0$$

$$S_{\perp} = \sqrt{n^2/4 - S_z^2}$$

$$n = |\psi_+|^2 + |\psi_-|^2$$

$$S_z = \frac{|\psi_+|^2 - |\psi_-|^2}{2}$$

$$\mu = \frac{\partial H}{\partial n}$$

To get the dispersion

$$i\frac{\partial \psi_+}{\partial t} = \frac{\delta H}{\delta \psi_+^*} = \left(T(i\nabla) - \Omega_z\right)\psi_+ + \left[\alpha_1 |\psi_+|^2 + \alpha_2 |\psi_-|^2\right]\psi_+$$

$$i\frac{\partial \psi_-}{\partial t} = \frac{\delta H}{\delta \psi_-^*} = \left(T(i\nabla) + \Omega_z\right)\psi_- + \left[\alpha_1 |\psi_-|^2 + \alpha_2 |\psi_+|^2\right]\psi_-$$

Spinor GP equations

Then represent the solution as

$$\vec{\psi}(\mathbf{r}, t) = \left(\sqrt{n}\vec{e} + \vec{A} e^{i(\mathbf{kr} - \omega t)} + \vec{B}^* e^{-i(\mathbf{kr} - \omega t)}\right) e^{-i\mu t}$$

$$\vec{e} = \begin{pmatrix} \sqrt{1/2 + s_z} \\ \sqrt{1/2 - s_z} \end{pmatrix}$$

And then linearize GP equations respect the small amplitudes of excitations ***A*** and ***B***

Dispersions of elementary excitations:

No magnetic field:
polarization is linear, $S_z=0$

$$\omega_{\pm}^2(k)=\omega_0^2(k)+(\alpha_1\pm\alpha_2)n\omega_0(k)$$

$$\mu=(\alpha_1+\alpha_2)n/2$$

Sound velocity depends on polarization $v_{s\pm}=\sqrt{\frac{(\alpha_1\pm\alpha_2)n}{2m}}$

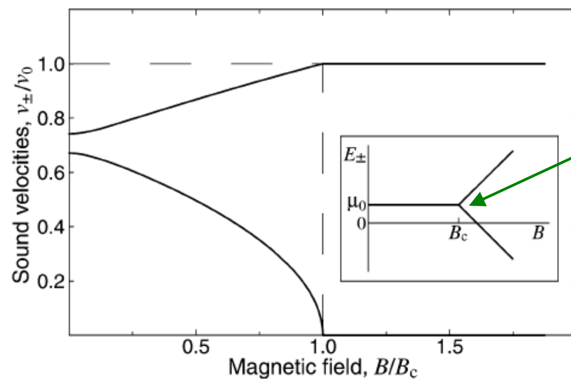
With magnetic field the polarization is elliptical

Critical magnetic field $\Omega_c=\frac{\alpha_1-\alpha_2}{2}n$

$\Omega<\Omega_c:\quad S_z=\Omega/(\alpha_1-\alpha_2),\quad \mu=(\alpha_1+\alpha_2)n/2$

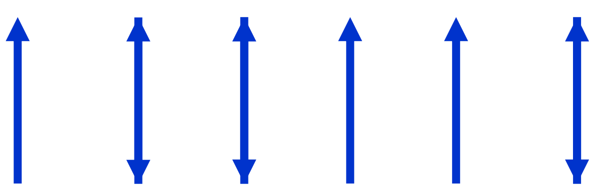
$\Omega>\Omega_c:\quad S_z=n/2,\quad \mu=\alpha_1 n-\Omega_z$

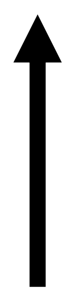
$$\omega_{\pm}^2=\omega_0^2+n\alpha_1\omega_0\pm\sqrt{(n\alpha_2)^2+(\alpha_1^2-\alpha_2^2)n^2\frac{\Omega^2}{\Omega_c^2}}\,\omega_0$$



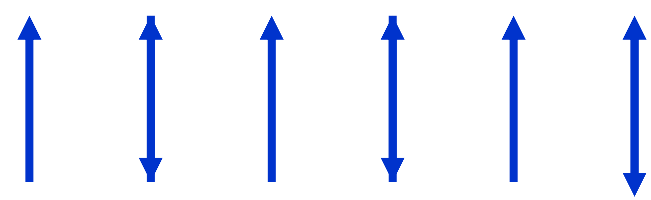
Full paramagnetic screening or “Spin Meissner Effect”

Effect of z-directed magnetic field:





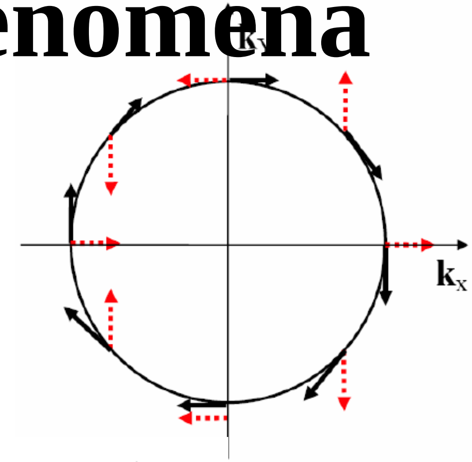
Effect of anizotropy of interaction:



Nonlinear polarization phenomena

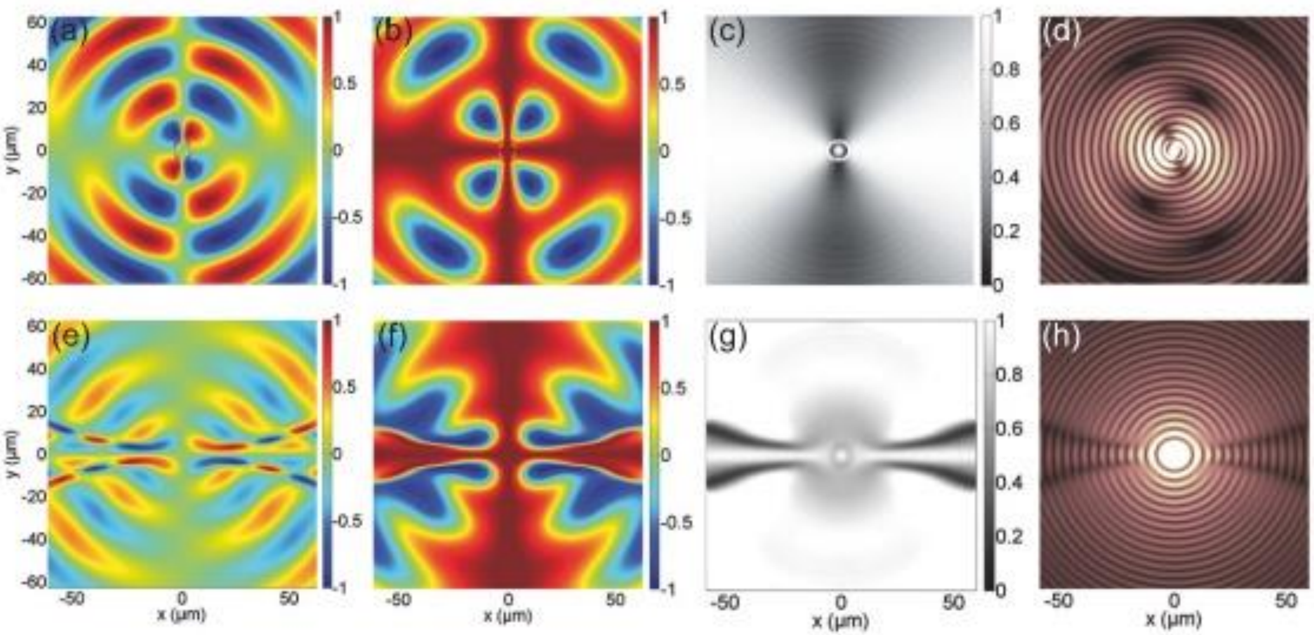
1. Bogoliubov renormalization of the dispersion
2. Superfluidity and supression of the Rayleigh scattering
3. Soliton- like propagation
4. Formation of vortices and half vortices
5. Parametric scattering
6. Optical Spin Hall Effect

$$H = \begin{pmatrix} H_0(k) & \beta(k_x - ik_y)^2 \\ \beta(k_x + ik_y)^2 & H_0(k) \end{pmatrix} = H_0(k)\mathbf{I} + \mathbf{\Omega}_{LT} \cdot \boldsymbol{\sigma}$$



$$\mathbf{\Omega}_{LT} = |\mathbf{\Omega}_{LT}|\left(\mathbf{e}_x \sin 2\varphi + \mathbf{e}_y \cos 2\varphi\right)$$

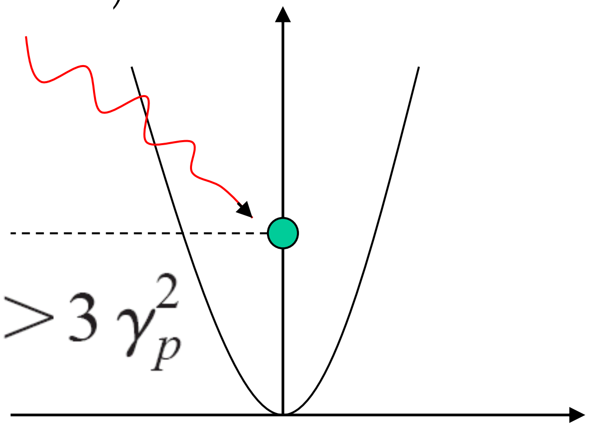
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix} = \begin{pmatrix} -\frac{\hbar^2}{2m^*}\nabla^2 - \mu + \alpha_1|\psi_+|^2 + \alpha_2|\psi_-|^2 & \beta\left(\frac{\partial}{\partial y} + i\frac{\partial}{\partial x}\right)^2 \\ \beta\left(\frac{\partial}{\partial y} - i\frac{\partial}{\partial x}\right)^2 & -\frac{\hbar^2}{2m^*}\nabla^2 - \mu + \alpha_1|\psi_-|^2 + \alpha_2|\psi_+|^2 \end{pmatrix}\begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix}$$



Nonlinear polarization phenomena under resonant pump: bistability

$$i\hbar\frac{\partial\psi_\pm}{\partial t} = -\hbar\gamma\psi_\pm + H_{LP}(-i\hbar\nabla)\psi_\pm + \left(\alpha_1|\psi_\pm|^2 + \alpha_2|\psi_\mp|^2\right)\psi_\pm + P_\pm e^{i\omega t} \qquad \alpha_1 \neq \alpha_2$$

Consider the case when pump is spatially homogenious and neglect the polarization. The dynamics of a scalar polariton field is described in this case by a following equation

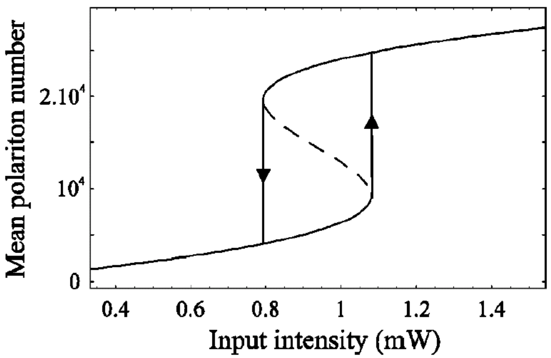


$$\delta_p^2 > 3\,\gamma_p^2$$

$$i\frac{\partial\psi}{\partial t} = (\delta_p - i\gamma)\psi + \alpha|\psi|^2\psi + \sqrt{\gamma}P$$

$$n\left[\gamma^2 + (\delta_p + \alpha n)^2\right] = \gamma I$$

$$n = |\psi|^2, I = |P|^2$$



This term leads to a variety of interesting nonlinerar effects including bistability

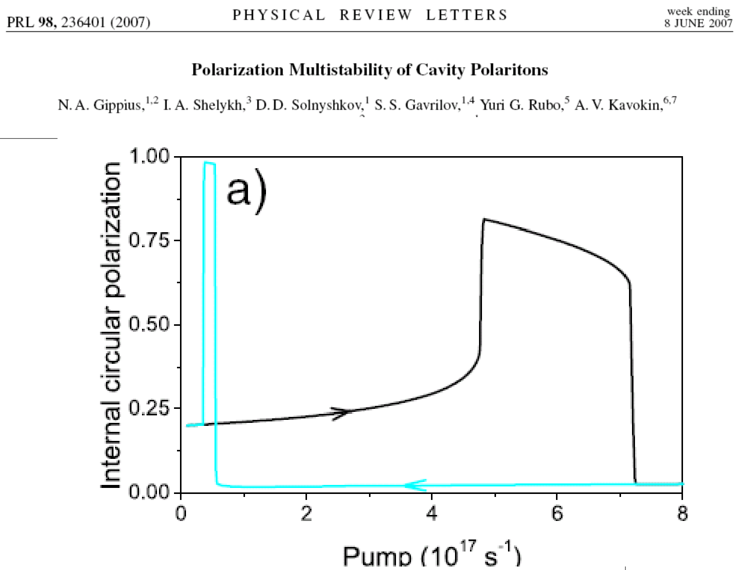
Including polarization: multistability

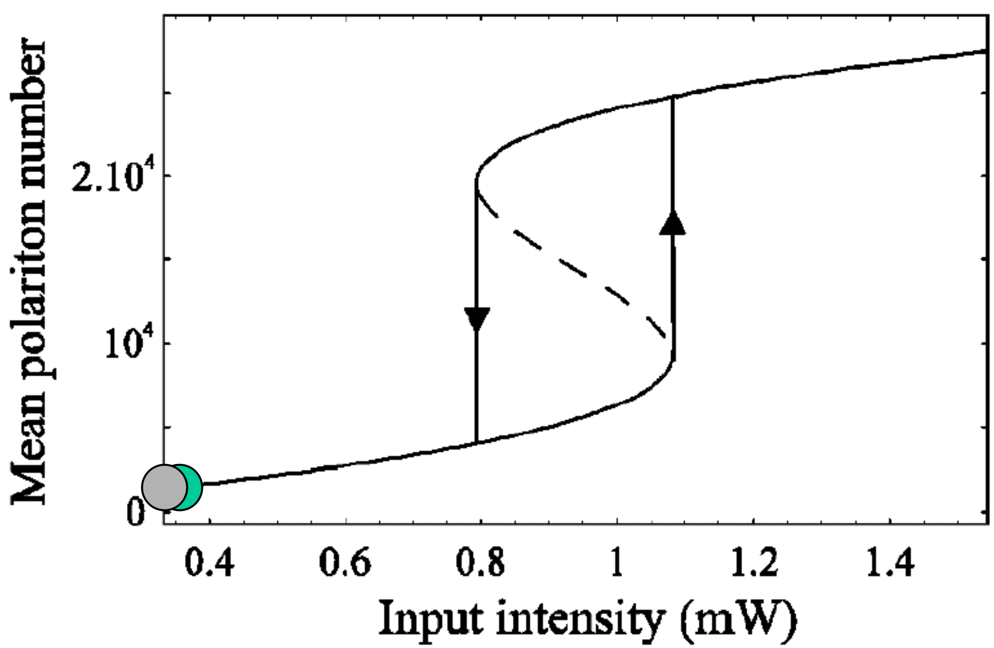
$$i\frac{\partial\psi_\sigma}{\partial t}=\left(\delta_p-i\gamma\right)\psi_\sigma+\left[\alpha_1\left|\psi_\sigma\right|^2+\alpha_2\left|\psi_{-\sigma}\right|^2\right]\psi_\sigma+\sqrt{\gamma}P_\sigma$$

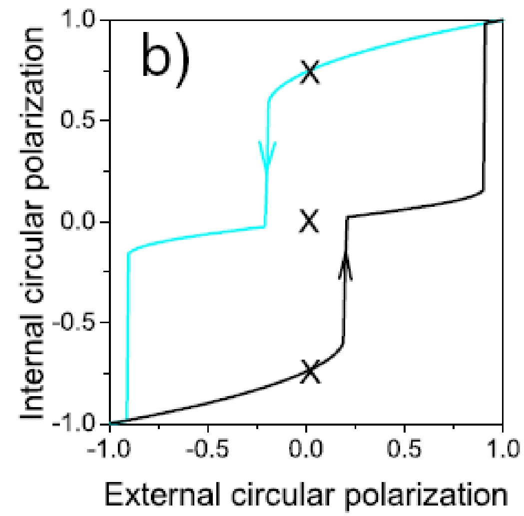
$$n_\sigma\left[\gamma^2+\left(\delta_p+\alpha_1 n_\sigma+\alpha_2 n_{-\sigma}\right)\right]=\gamma I_\sigma$$

σ⁺ σ⁻

Low circular polarization / High circular polarization





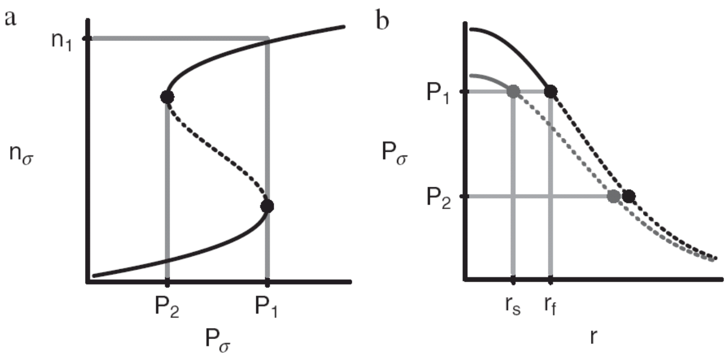


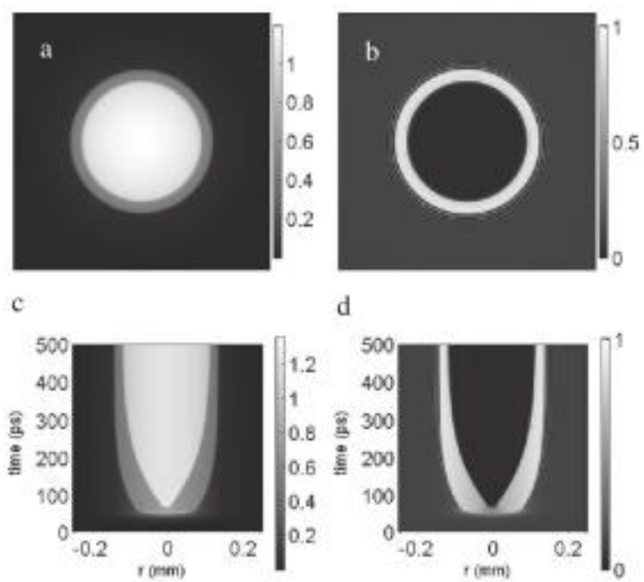
Effects of multistability in the real space

PRL **100,** 116401 (2008) PHYSICAL REVIEW LETTERS week ending 21 MARCH 2008

Spin Rings in Semiconductor Microcavities

I. A. Shelykh,[1,2] T. C. H. Liew,[1,3] and A. V. Kavokin[3,4]





RAPID COMMUNICATIONS
PHYSICAL REVIEW B **80**, 161303(R) (2009)

Polarization phenomena in resonantly pumped disordered semiconductor microcavities

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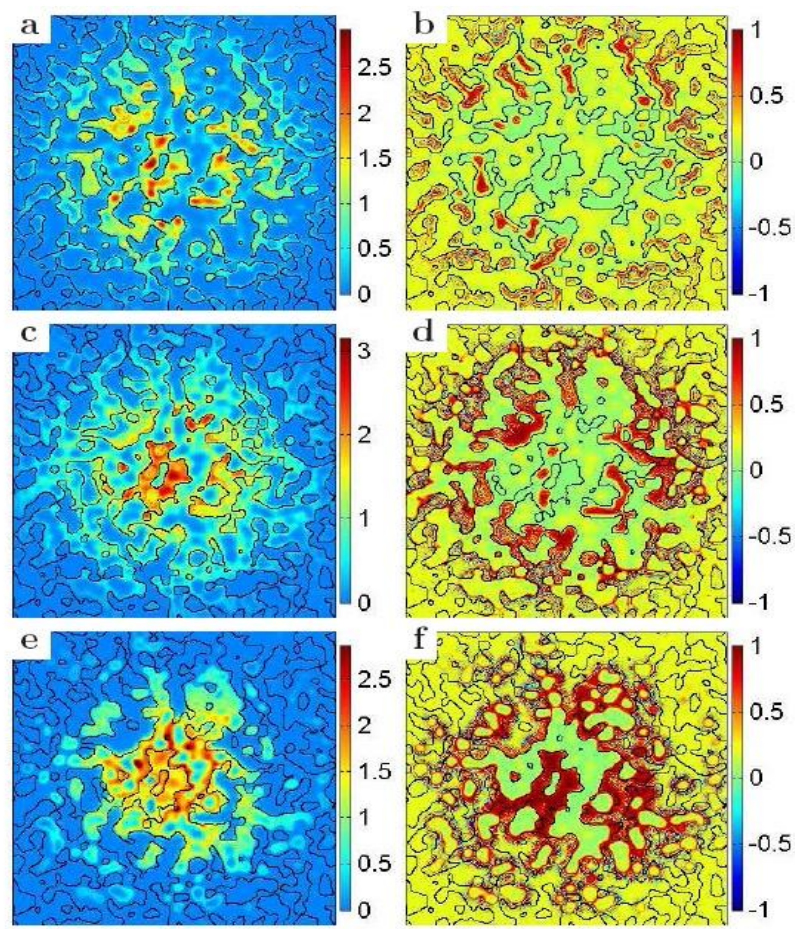


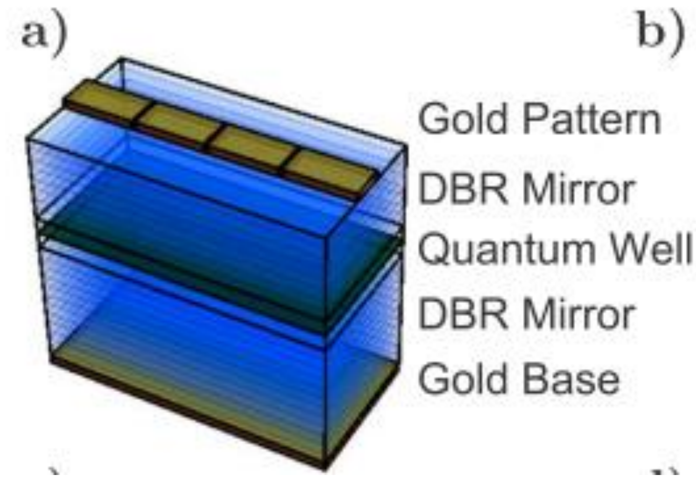
Polariton neurons and integrated circuits

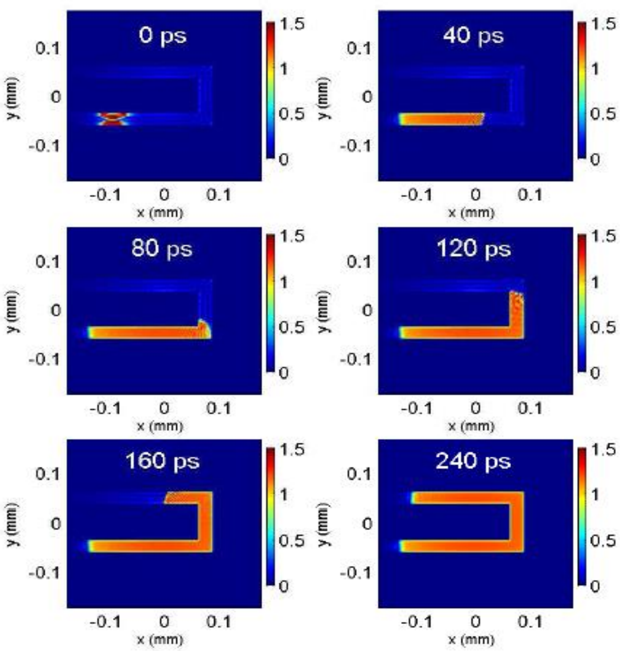
PRL 101, 016402 (2008) PHYSICAL REVIEW LETTERS week ending 4 JULY 2008

Optical Circuits Based on Polariton Neurons in Semiconductor Microcavities

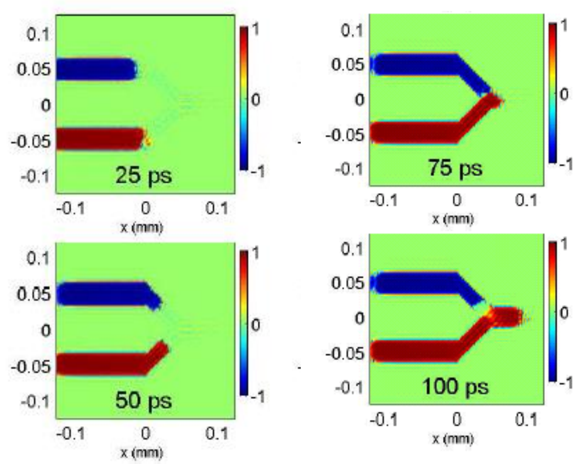
T. C. H. Liew,[1,2] A. V. Kavokin,[1,3] and I. A. Shelykh[2,4]

The propagation of the "domain wall" between the regions of strong and weak circular polarizations in a system of spatially confined polaritons





Background: bias towards σ+. Pulses: σ- and σ+



After the junction only σ+ continue

The system behaves as OR logic gate

Real space density matrix approach

What we need to determine the dynamics of the polariton system in the real space?

The dynamic equation for $$n(\mathbf{r},t)=\lim_{\mathbf{r}\to\mathbf{r}'}\rho(\mathbf{r},\mathbf{r}',t)=\lim_{\mathbf{r}\to\mathbf{r}'}\left\langle\psi^{+}(\mathbf{r},t)\psi(\mathbf{r}',t)\right\rangle$$

The density matrix in the real space can be found as a Fourier transfer of the density matrix in the reciprocal space

$$\rho(\mathbf{r},\mathbf{r},t)=\left(\frac{A}{4\pi^2}\right)\lim_{\mathbf{r}\to\mathbf{r}'}\int e^{i(\mathbf{kr}-\mathbf{k}'\mathbf{r}')}\rho(\mathbf{k},\mathbf{k}',t)d\mathbf{k}d\mathbf{k}',$$

$$\rho(\mathbf{k},\mathbf{k}',t)=\left\langle a_{\mathbf{k}}^{+}a_{\mathbf{k}'}\right\rangle$$

The diagonal matrix elements of the density matrix in k- representation are occupation numbers. But for description of dynamics of spatially inhomogenipous system we need off- diagonal elements as well

Boltzmann limit:

$$\rho(\mathbf{k},\mathbf{k}',t)=n_{\mathbf{k}}\delta\left(\mathbf{k}-\mathbf{k}'\right),$$

$$\rho(\mathbf{r},\mathbf{r},t)=const$$

Fully coherent limit:

$$\rho(\mathbf{k},\mathbf{k}',t)=\left\langle a_{\mathbf{k}}\right\rangle^{*}\left\langle a_{\mathbf{k}'}\right\rangle$$

$$\rho(\mathbf{r},\mathbf{r},t)=\left\langle\psi(\mathbf{r},t)\right\rangle^{*}\left\langle\psi(\mathbf{r},t)\right\rangle$$

How it is possible to write the dynamic equations for both occupation numbers and coherencies accounting for the non- energy conserving processes of the phonon scattering?

Real space density matrix approach

Divide the total Hamiltonian of the system into "coherent part" H_1 and "incoherent part" H_2

$$H = H_1 + H_2$$

$$H_1 = \sum_{\mathbf{k}} E_{\mathbf{k}} a^{+}_{\mathbf{k}} a_{\mathbf{k}} + \frac{U}{2} \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{p}} a^{+}_{\mathbf{k}_1} a^{+}_{\mathbf{k}_2} a_{\mathbf{k}_1+\mathbf{p}} a_{\mathbf{k}_2-\mathbf{p}}$$

$$i\partial_t^{(1)}\rho = \left[H_1; \rho\right]$$

$$\widehat{H}_2(t) = H^{-}(t) + H^{+}(t) = \qquad (\
= \sum_{\mathbf{k},\mathbf{q}} D(\mathbf{q}) e^{i(E_{\mathbf{k}+\mathbf{q}} - E_{\mathbf{k}})t} a^{+}_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}} \left(b_{\mathbf{q}} e^{-i\omega_{\mathbf{q}} t} + b^{+}_{-\mathbf{q}} e^{i\omega_{\mathbf{q}} t}\right)$$

$$\partial_t^{(2)}\rho = -\int_{-\infty}^{t} dt' \left[H_2(t); \left[H_2(t'); \rho(t')\right]\right] =$$

$$= -\int_{-\infty}^{t} dt' \left[H_2(t); \left[H_2(t'); \rho(t)\right]\right] =$$

$$= \delta_{\Delta E} \left\{ 2\left(H^{+}\rho H^{-} + H^{-}\rho H^{+}\right) - \left[H^{+}H^{-} + H^{-}H^{+}; \rho\right]_{+} \right\}$$

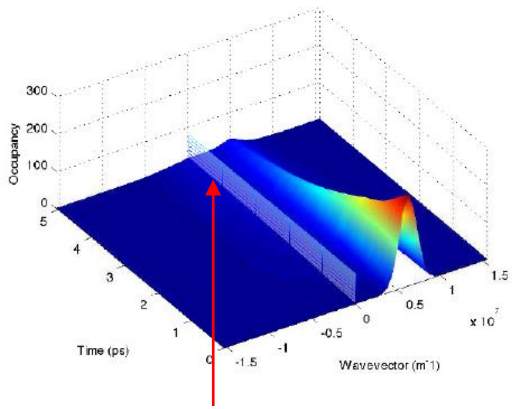
$$\{\partial_t \rho(\mathbf{k},\mathbf{k})\}^{(1)} = 2U \sum_{\mathbf{k}_1,\mathbf{p}} \mathrm{Im}\left\{\rho(\mathbf{k}_1, \mathbf{k}_1 - \mathbf{p}) \rho(\mathbf{k}, \mathbf{k} + \mathbf{p})\right\} \qquad ((\
$$

$$\{\partial_t \rho(\mathbf{k},\mathbf{k}')\}^{(1)} = i(E_{\mathbf{k}} - E'_{\mathbf{k}})\rho(\mathbf{k},\mathbf{k}') + \qquad ('\
+ iU \sum_{\mathbf{k}_1,\mathbf{p}} \rho(\mathbf{k}_1, \mathbf{k}_1 - \mathbf{p}) \left[\rho(\mathbf{k} - \mathbf{p}, \mathbf{k}') - \rho(\mathbf{k}, \mathbf{k}' + \mathbf{p})\right]$$

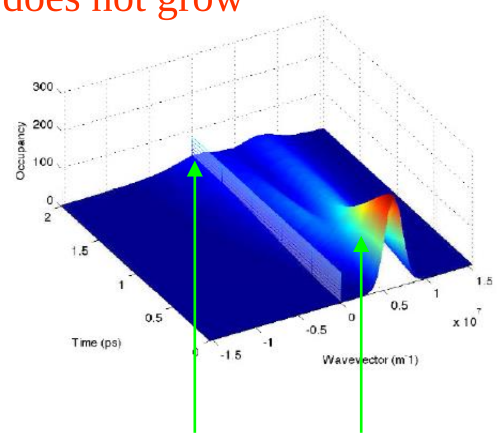
$$\{\partial_t \rho(\mathbf{k},\mathbf{k}')\}^{(2)} = \rho(\mathbf{k},\mathbf{k}') \times \qquad (12)\
\times \left\{ \sum_{\mathbf{q}', E_{\mathbf{k}} < E_{\mathbf{k}+\mathbf{q}'}} W(\mathbf{q}') \left[\rho(\mathbf{k}+\mathbf{q}', \mathbf{k}+\mathbf{q}') - n^{ph}_{\mathbf{q}'}\right] + \right.\
+ \sum_{\mathbf{q}', E_{\mathbf{k}} > E_{\mathbf{k}+\mathbf{q}'}} W(\mathbf{q}') \left[-\rho(\mathbf{k}+\mathbf{q}', \mathbf{k}+\mathbf{q}') - n^{ph}_{\mathbf{q}'} - 1\right] +\
+ \sum_{\mathbf{q}', E'_{\mathbf{k}} < E_{\mathbf{k}'+\mathbf{q}'}} W(\mathbf{q}') \left[\rho(\mathbf{k}' + \mathbf{q}', \mathbf{k}' + \mathbf{q}') - n^{ph}_{\mathbf{q}'}\right] +\
\left. + \sum_{\mathbf{q}', E'_{\mathbf{k}} > E_{\mathbf{k}'+\mathbf{q}'}} W(\mathbf{q}') \left[-\rho(\mathbf{k}' + \mathbf{q}', \mathbf{k}' + \mathbf{q}') - n^{ph}_{\mathbf{q}'} - 1\right] \right\}$$

Analog of Gross- Pitaevskii equation written for the density matrix

Dynamics of the system at finite temperature

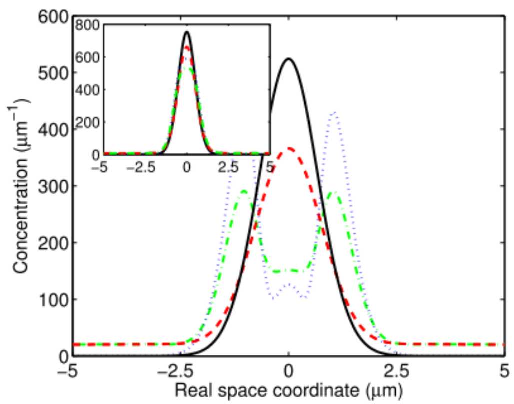


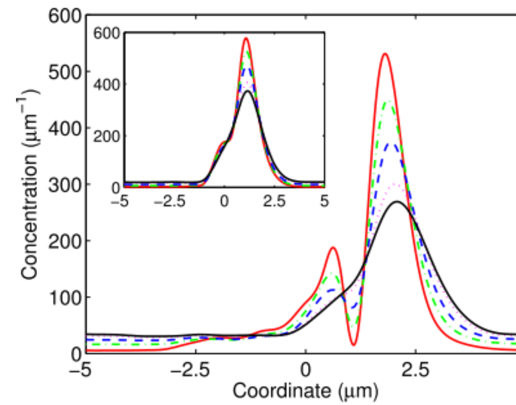
Low pump: bottleneck effect, occupancy of k=0 state almost does not grow



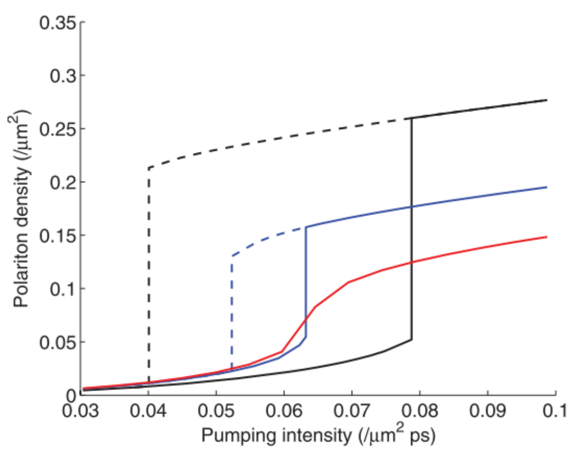
High pump: overcoming of the bottleneck effect, additional maximum of the population appears at higher k

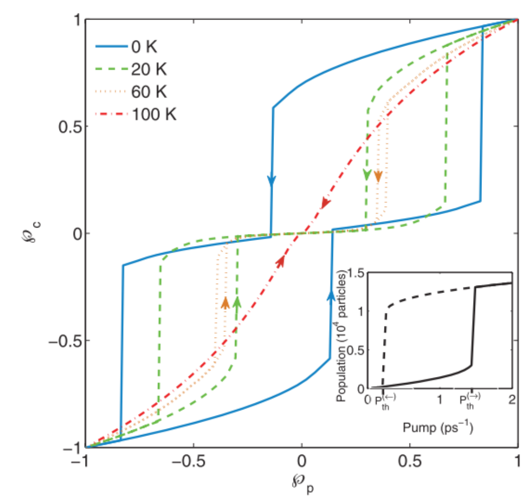
Real space dynamics





Effects of multistability

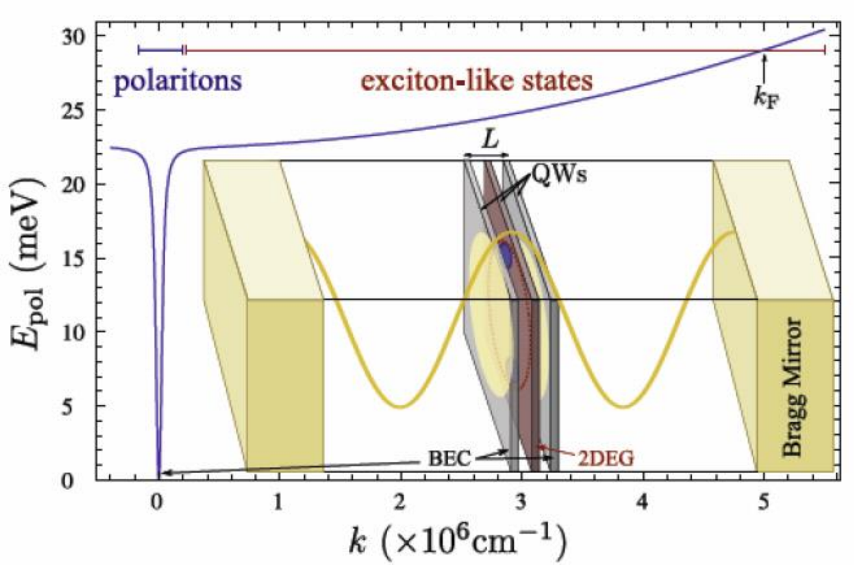




Polariton- mediated superconductivity

BCS Mechanism: Formation of the Cooper pairs due to phonon- mediated attraction. Can phonon be replaced by other quasiparticle???

Geometry of the device:



PRL 104, 106402 (2010) PHYSICAL REVIEW LETTERS week ending 12 MARCH 2010

Exciton-Polariton Mediated Superconductivity

Fabrice P. Laussy,[1] Alexey V. Kavokin,[1,2] and Ivan A. Shelykh[3,4]

$$H = \sum_{\mathbf{k}} \left[E_{\mathrm{el}}(\mathbf{k}) \sigma^\dagger_{\mathbf{k}} \sigma_{\mathbf{k}} + E_{\mathrm{pol}}(\mathbf{k}) a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \right] + \\ + \sum_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}} \Big[V_{\mathrm{C}}(\mathbf{q}) \sigma^\dagger_{\mathbf{k}_1+\mathbf{q}} \sigma^\dagger_{\mathbf{k}_2-\mathbf{q}} \sigma_{\mathbf{k}_1} \sigma_{\mathbf{k}_2}, \\ + X V_{\mathrm{X}}(\mathbf{q}) \sigma^\dagger_{\mathbf{k}_1} \sigma_{\mathbf{k}_1+\mathbf{q}} a^\dagger_{\mathbf{k}_2+\mathbf{q}} a_{\mathbf{k}_2} + U a^\dagger_{\mathbf{k}_1} a^\dagger_{\mathbf{k}_2+\mathbf{q}} a_{\mathbf{k}_1+\mathbf{q}} a_{\mathbf{k}_2} \Big]$$

$$V_{\mathrm{X}}(\mathbf{q}) = \frac{16 e^2}{A \epsilon a_{\mathrm{B}}^3} \frac{\pi e^{-|\mathbf{q}|L/2}}{|\mathbf{q}| + \kappa_{\mathbf{q}}} \times \\ \left\{ \frac{1}{\beta_h^2} \frac{1}{\left(|\mathbf{q}|^2 + \frac{4}{a_{\mathrm{B}}^2 \beta_h^2}\right)^{3/2}} - \frac{1}{\beta_e^2} \frac{1}{\left(|\mathbf{q}|^2 + \frac{4}{a_{\mathrm{B}}^2 \beta_e^2}\right)^{3/2}} \right\}$$

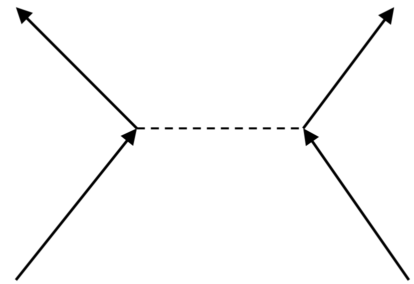
$$H = \sum_{\mathbf{k}} E_{el}(\mathbf{k}) \sigma^+_{\mathbf{k}} \sigma_{\mathbf{k}} + \sum_{\mathbf{k}} E_{bog}(\mathbf{k}) b^+_{\mathbf{k}} b_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{q}} M(\mathbf{q}) \sigma^+_{\mathbf{k}} \sigma_{\mathbf{k}-\mathbf{q}} \left(b^+_{-\mathbf{q}} + b_{\mathbf{q}} \right)$$

Frolich type Hamiltonian

Using mean field approximation and Bogoliubov transformation

$$a^\dagger_{\mathbf{k}_1+\mathbf{q}} a_{\mathbf{k}_1} \approx \langle a^\dagger_{\mathbf{k}_1+\mathbf{q}} \rangle a_{\mathbf{k}_1} + a^\dagger_{\mathbf{k}_1+\mathbf{q}} \langle a_{\mathbf{k}_1} \rangle$$

$$\langle a_{\mathbf{k}} \rangle \approx \sqrt{N_0 A}\, \delta_{\mathbf{k},\mathbf{0}}$$



One reduces Hamiltonian to:

Frolich term: gives Bogolon-mediated attraction proportional to N_0

$$H = \sum_{\mathbf{k}} E_{\text{el}}(\mathbf{k}) \sigma^\dagger_{\mathbf{k}} \sigma_{\mathbf{k}} + \sum_{\mathbf{k}} E_{\text{bog}}(\mathbf{k}) b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + + H_{\text{C}} + \sum_{\mathbf{k},\mathbf{q}} M(\mathbf{q}) \sigma^\dagger_{\mathbf{k}} \sigma_{\mathbf{k}+\mathbf{q}} (b^\dagger_{-\mathbf{q}} + b_{\mathbf{q}})$$

$$V_{\text{A}}(\mathbf{q}, \omega) = \frac{2M(\mathbf{q})^2 E_{\text{bog}}(\mathbf{q})}{(\hbar\omega)^2 - E_{\text{bog}}(\mathbf{q})^2}$$

$$E_{\text{bog}}(\mathbf{k}) = \sqrt{\tilde{E}_{\text{pol}}(\mathbf{k})(\tilde{E}_{\text{pol}}(\mathbf{k}) + 2UN_0 A)}$$

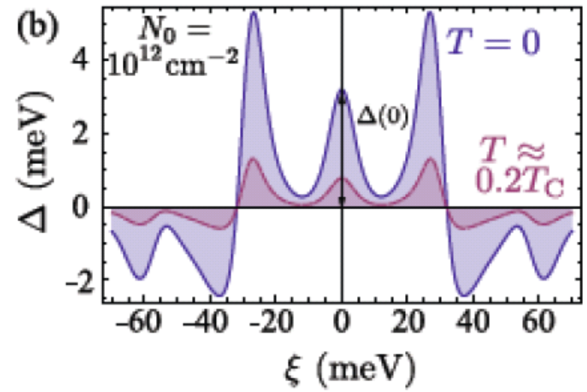
$$M(\mathbf{q}) = \sqrt{N_0 A}\, X V_{\text{X}}(\mathbf{q}) \sqrt{\frac{E_{\text{bog}}(\mathbf{q}) - \tilde{E}_{\text{pol}}(\mathbf{q})}{2UN_0 A - E_{\text{bog}}(\mathbf{q}) + \tilde{E}_{\text{pol}}(\mathbf{q})}}$$

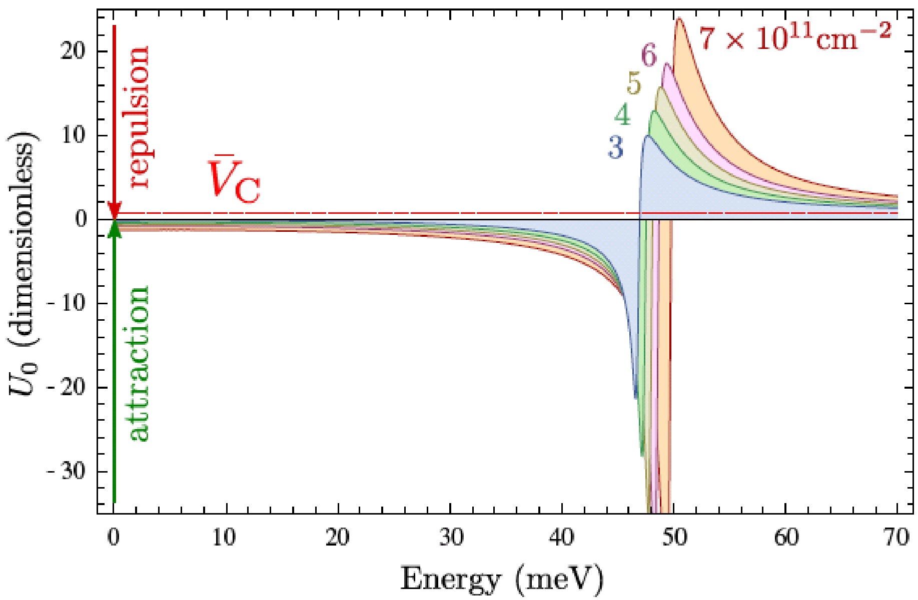
Gap and critical temperature

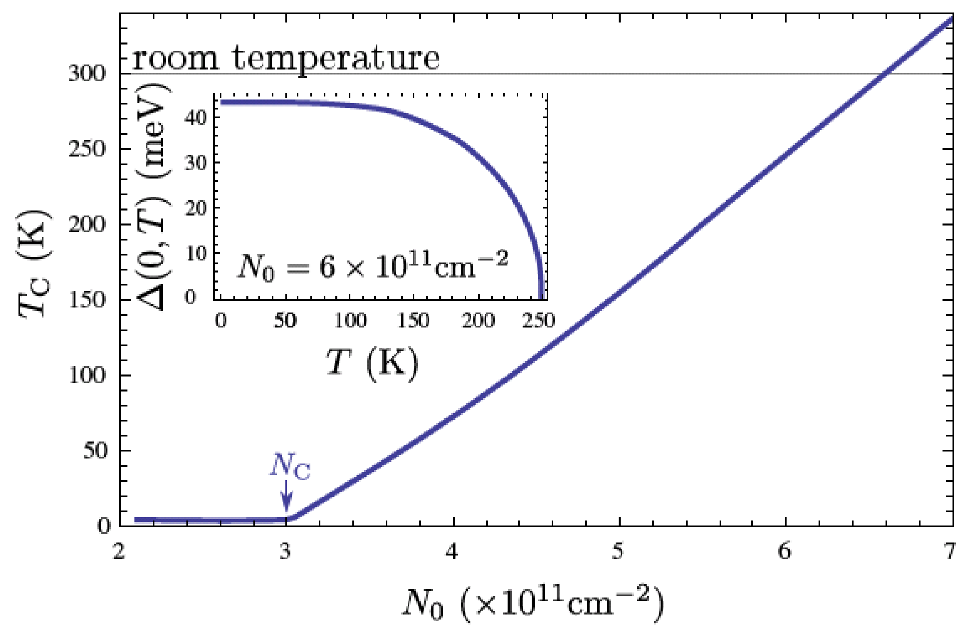
$$\Delta(\xi, T) = -\int_{-\infty}^{+\infty} \frac{U_0(\xi - \xi')\Delta(\xi', T)\tanh(E/2k_{\mathrm{B}}T)}{2E} d\xi'$$

$$E = \sqrt{\Delta(\xi', T)^2 + \xi'^2}$$

$$U_0(\omega) = \frac{A\mathcal{N}}{2\pi}\int_0^{2\pi} [V_{\mathrm{A}}(q,\omega) + V_{\mathrm{C}}(q)]d\theta$$







Conclusions

1°) Semiconductor microcavity is a unique laboratory for study of different collective phanomena

2°) Polaritons exhibit high- temperature BEC and superfluidity

3°) Polariton-polariton interactons are spin-anizotropic which leads to a variety of polarization- dependent nonlinear effects such as polarization multistability

4°) In hybrid polariton- electron structures polariton-mediated superconductivity can appear.