Quantum Optics

Example of a problem set

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Submission deadline: 3rd of December

1. **Density matrix of thermal light**

   The density matrix of a system, which is in the thermodynamical equilibrium with reservoir of the temperature T can be expressed as:

$$\hat{\rho} = \frac{\exp\left(-\frac{\hat{H}}{kT}\right)}{Tr\left(\exp\left(-\frac{\hat{H}}{kT}\right)\right)}$$

   a) (3 pts) Construct the density matrix for a **single-mode** field in thermal equilibrium.

   b) (3 pts) Compute the second order correlation function $g^{(2)}(0)$ for thermal light

$$g^{(2)} = \frac{\langle a^{+}(0)a^{+}(\tau)a(\tau)a(0)\rangle}{\langle a^{+}(0)a(0)\rangle^{2}}$$

   c) (2 pts) Construct the density matrix for a **multi-mode** field in thermal equilibrium.

2. (4 pt) **Coherent states in position space.**

   Find the eigenfunctions of a coherent state $|\alpha\rangle$ in the **position** space (either analytically or numerically).

3. (6 pts) **Random phase signal**

   Imagine you have a field signal, which is given by: $E^{(+)} \sim e^{-i\omega t}e^{-i\varphi(t)}$. Find the spectrum of the signal

$$I(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} g^{(1)}(\tau)\cos(\omega\tau),$$

   assuming that $\varphi(t)$ is a random Gaussian function, with correlation function $S_{\varphi}(\omega) = 1/\omega^{2}$:

$$\langle\varphi(t)\varphi(t+\tau)\rangle = \frac{1}{\pi}\int_{0}^{+\infty}\frac{1}{\omega^{2}}\cos(\omega\tau)d\omega.$$

4. (6 pts) **Nonlinear cavity**

One of the common models of light-atom interaction in a cavity can be described by the Hamiltonian :

$$H = \hbar\omega_0\sigma_z + \hbar\omega a^+ a + \hbar g(\sqrt{a^+ a}a^+\sigma_- + \sigma_+ a\sqrt{a^+ a}),$$

where interaction depends on the intensity. Compute the inverse population at the timescale of Rabi oscillations, considering initial state of photons as coherent.