Homework 4. Topological insulators.

October 26, 2018

1. Consider the full Graphene Hamiltonian with lattice staggered potential (i.e. not in the vicinity of the Dirac point but valid in the whole Brillouin zone). Staggered potential changes the on-site energy of the sublattice A with respect to sublattice B by Δ. Use maple to plot the Berry curvature for the two bands in the whole Brilloin zone. Without the integration can you "guess" what is the Chern number? Obtain the effective Hamiltonians in the vicinity of K and K'. What is the result of the action of time-reversal operator on the Hamiltonian $H(K+k)$? $H(K'+k)$?

2. Consider the simplest Hamiltonian with chiral edge state. The 2d system lies in (x,y) plane and is described by the hamiltonian
$$H = \sigma \mathbf{p} + \sigma(z) m(x), \tag{1}$$
where $m(x)$ is a smooth function such that $m(0) = 0$, $m(-\infty) = -1, m(+\infty) = 1$.. What are the eigenvalues and eigenvector of the bound state. Does the dispersion depend on the specific value of $m(z)$?

3. Consider the Simplest 2d topological insulator introduced by Bernevig, Hughes and Zhang. In this 2d model, the lattice is square, and each lattice cite has 4 degrees of freedom $|s\uparrow\rangle, |s\downarrow\rangle, |p\uparrow\rangle, |p\downarrow\rangle$. Only tight binding hamiltonian is considered. We consider now spin-orbit coupling, thus there is only coupling between the same spin projections. couplings t_{ss} and t_{pp} are real and have opposite signs (t_{pp})¡0. $t_{sp} = t^*_{ps}$ are complex and depend on the direction of hopping. The full Hamitonian is
$$H = H_0 + H_1, \tag{2}$$
$$H_0 = \sum_{\mathbf{R},\sigma=\uparrow,\downarrow} \epsilon_s |Rs\sigma\rangle|\langle Rs\sigma| + \epsilon_p |Rs\sigma\rangle\langle Rp\sigma|, \tag{3}$$
$$H_1 = -\sum_{R\sigma} \sum_{\mu=\pm x,\pm y} t_{ss}|R+\mathbf{a}_m s\sigma\rangle\langle Rs\sigma| - |t_{pp}||R+\mathbf{a}_m p\sigma\rangle\langle Rp\sigma| + [t_{sp,\mu}|R+\mathbf{a}_m s\sigma\rangle\langle Rp\sigma| + h.c.] \tag{4}$$
where $t_{sp,\pm x} = \pm 1, t_{sp,\pm y} = \pm i$. let $|t_{ss}| = |t_{pp}| = t_0$. Let also $\epsilon_s = e_0 - \delta$, $\epsilon_p = e_0 + \delta$. Plot the bulk 3d dispersion for the cases $\delta < 4t_0, \delta = 4t_0, \delta > 4t_0$. Project 3d dispersion to the k_x, E plane. Then make a stripe of a finite length - of 4 unit cells along y. Plot the dispersion equations $E(kx)$. For which of the three cases states in the gap are observed. Obtain the eigenvectors of the gap states - check that they are ortothogonal. You may observe that the edge states are slightly gapped - what is the source of the gap?.